Appendix: A DSGE evaluation of fiscal rules in Portugal: Notes

Francesco Franco

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Details

The model used for the stochastic simulations is a small open economy NKM in a monetary union. It features

1. Two types of consuming households (Gali 2007):
   (a) a Ricardian intertemporally optimizing households
   (b) a liquidity constrained, non-Ricardian hand-to-mouth households.
2. Involuntary unemployment (Gali 2008)
3. Nominal rigidities: wage rigidity (Erceg, Henderson, and Levin 2000), Nominal price rigidity (Calvo 1983)
4. Real rigidities: Investment adjustment costs (Christiano Eichembaum Evans 2005), Capital Utilization (Smets and Wouters 2003), habit consumption
5. Final sectors in consumption and investment goods that use imported inputs.
6. Fiscal rules: nominal balance, structural balance, expenditure growth, debt break.

To evaluate the rules through stochastic simulations. The simulations are based onnthe historical distribution of shocks over 1995-2015. The results show the counterfactual historical performance of these rules in terms of achieving debt sustainability.

Households

Following Gali (2008) each household member is characterized by the quality of its labour services $j \in [0,1]$ and by his disutlity of labor $h \in [0,1]$. Therefore the model assumes a representative household with a continuum of members represented by the unit square and indexed by a pair $(h,j) \in [0,1] \times [0,1]$. Individual utility is given by

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{\left(\bar{C}_t(h,j)\right)^{1-\sigma} - 1}{1-\sigma} - \chi_t \mathbb{I}_t(h,j) h^{\varphi} \right) Z_t$$

where individual consumption exhibit internal habit $\bar{C}_t(h,j) = \tilde{C}_t(h,j) - \mathfrak{h}\tilde{C}_{t-1}(h,j)$ and depends on private and public consumption: $\tilde{C}_t(h,j) = C_t(h,j) + \alpha_G G_t(h,j)$. To avoid keeping track of households members heterogeneity we follow Merz (1996) and assume that households members can trade state contingent securities that permits to equate marginal ultility of consumption across households members. Separability of preferences ensures that households have identical consumption and investment plans. We can therefore write the household utility as the integral over its members' utilites:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{\bar{C}_t^{1-\sigma} - 1}{1-\sigma} - \chi_t \int_0^1 \frac{\mathcal{N}_t(h)^{1+\varphi}}{1+\varphi} dh \right) Z_t.$$

A fraction μ of the working household members saves while a fraction $1-\mu$ does not. The savers maximizes their utility subject to the following dynamic budget constraint:

$$P_{C,t}\left(1+\tau_{C,t}\right) C_t^R + P_{I,t} I_t + \sum_{k \in J} \Psi_{k,t} B_{k,t} + P_{C,t} \mathcal{C}\left(\upsilon_t\right) K_{t-1} = \sum_{k \in J} B_{k,t-1} + \left(1-\tau_{N,t}\right) \int_0^1 W_t(j) N_t(j) dj + \left(1-\tau_{K,t}\right) R_{K,t} \upsilon_t K_{t-1} + T_t^R + \Pi_t$$

where $J = \{F, H, G\}$ indicates the type of financial asset inviduals use. We allow for the existence of foreign bonds, domestic bonds and and domestic governments bonds.

Capital accumulates

$$K_t = (1-\delta) K_{t-1} + \mathcal{D}_t F\left(I_t, I_{t-1}\right)$$

where $F(I_t, I_{t-1}) = \left(1 - S\left(\frac{I_t}{I_{t-1}}\right)\right) I_t$, with $S(1) = 0, S'(1) = 0, S''(1) = s > 0$ and $\mathcal{D}_t$ is an investment specific technology shock. The non savers, denoted by a superscript NR, maximize their utility subject to the static budget constraint:

$$P_{C,t}\left(1+\tau_t^C\right) C_t^{NR} = \left(1-\tau_t^N\right) \int_0^1 W_t(j) N_t(j) \, dj + T_t^{NR}$$

The households members would want to participate in the labor force according to the following condition:

$$\frac{W_t^*(j)}{P_{C,t}} = \frac{\chi_t L(j)^{\phi} Z_t}{\Lambda_t \left(1-\tau_t^N\right)}$$

so that the unemployed (involuntary) in the houshold are

$$U(j) = L(j) - N(j).$$

The unemployed receive an unemployment benefit which is a fraction τ_t^U of the market wage corresponding to their skill.

$$P_{C,t}\left(1+\tau_t^C\right) C_t^U = \tau_t^U \int_0^1 W_t(j) U_t(j) \, dj$$

Consumption-Saving choices

The marginal utilty of consumption for the savers is

$$\Lambda_t = \left(1+\tau_t^C\right)^{-1} \left[\left(\widetilde{C}_t\right)^{-\sigma} Z_t - \beta \mathfrak{h} E_t \left(\widetilde{C}_{t+1}\right)^{-\sigma} Z_{t+1}\right].$$

The savers' Euler equation corresponding to the demand of asset k

$$\Lambda_t \Psi_{k,t} = \beta E_t \left[\Lambda_{t+1} \frac{P_{C,t}}{P_{C,t+1}}\right].$$

Investment decisions

Households optimal capital decision is where $Q_t \Lambda_t$ is the lagrange multiplier on the investment accumulation equation

$$Q_t = \beta E_t \left[\frac{\Lambda_{t+1}}{\Lambda_t} \left((1-\tau_{K,t+1}) \frac{R_{K,t+1}^K}{P_{C,t+1}} \upsilon_{t+1} - \mathcal{C}(\upsilon_{t+1}) + (1-\delta) Q_{t+1}\right)\right]$$

their optimal investment decision is

$$\frac{P_{I,t}}{P_{C,t}} = Q_t \mathcal{D}_t \left\{1 - S\left(\frac{I_t}{I_{t-1}}\right) - \frac{I_t}{I_{t-1}} S'\left(\frac{I_t}{I_{t-1}}\right)\right\} + \beta E_t \left[\mathcal{D}_{t+1} \frac{\Lambda_{t+1}}{\Lambda_t} Q_{t+1} \left\{S'\left(\frac{I_{t+1}}{I_t}\right)\right\} \left(\frac{I_{t+1}}{I_t}\right)^2\right]$$

and the optimal capital utilization

$$\mathcal{C}'(\upsilon_t) = (1-\tau_{K,t}) \frac{R_{K,t}}{P_{C,t}}$$

Labor supply and wage settings

Calvo wage setting that also index to inflation if cannot reoptimize

$$W_{t+k|t}(j) = W_t^*(j) \mathcal{I}_{t+k|t}^W$$

where $\mathcal{I}_{t+k|t}^W$ is an indexing rule. Labor demand comes from firm minimization

$$N_t(i) = \left[\int_0^1 N_t(i,j)^{1-\frac{1}{\epsilon_w}} dj\right]^{\frac{\epsilon_w}{\epsilon_w - 1}}$$

$$N_{t+k|t}(j) = \left(\frac{W_t^*(j) \mathcal{I}_{t+k|t}^W}{W_{t+k}}\right)^{-\epsilon_w} \int_0^1 N_{t+k}(i) di$$

the optimal wage is

$$\left(\frac{W_t^*}{P_{C,t}}\right)^{1+\phi\epsilon_w} = \frac{\epsilon_w}{\epsilon_w - 1} \frac{\mathcal{F^W}_t}{\mathcal{G^W}_t}$$

where $\mathcal{F^W}_t = \chi_t \left(\frac{W_t}{P_{C,t}}\right)^{(1+\phi)\epsilon_w} \mathcal{N}_t^{1+\phi} Z_t + \beta\theta_w E_t \left(\frac{P_C}{P_{C,t+1}} \mathcal{I}^W_{t+1|t}\right)^{-\epsilon_w(1+\phi)} \mathcal{F^W}_{t+1}$ and $\mathcal{G^W}_t = \Lambda_t \left(\left(1-\tau_t^N\right) N_t\right) \left(\frac{W_t}{P_{C,t}}\right)^{\epsilon_w} + \beta\theta_w E_t \mathcal{G^W}_{t+1} \Big($

Notice that if wages are felxible we get the usual:

$$\left(\frac{W_t^*}{P_{C,t}}\right)^{flex} = \frac{\epsilon_w}{\epsilon_w - 1} \frac{\chi_t \mathcal{N}_t^{\phi} Z_t}{\Lambda_t \left(1-\tau_t^N\right)}$$

The aggregate wage aggregate wage index is

$$W_t = \left[\int_0^1 W_t(j)^{1-\epsilon_w} dj\right]^{\frac{1}{1-\epsilon_w}}.$$

Intermediate Firms

Technology is identical across firms which are indexed by i

$$Y_t\left(i\right) = \bar{K}_t\left(i\right)^{\alpha} \left(A_t N_t(i)\right)^{1-\alpha},$$

with effective capital:

$$\bar{K}_t(i) = v_t K_{t-1}(i).$$

The labor input is defined by

$$N_t(i) = \left[\int_0^1 N_t(i,j)^{\frac{\epsilon_w - 1}{\epsilon_w}} dj\right]^{\frac{\epsilon_w}{\epsilon_w - 1}}.$$

Profits are

$$\Pi_t(i) = P_{H,t}(i) Y_t\left(i\right) - \left(1+\tau_{Pay,t}\right) W_t N_t(i) - \left(1+\tau_{R,t}\right) R_{K,t} \bar{K}_t(i)$$

define the marginal costs

$$MC_{t+s|t} = \widehat{\alpha} \frac{\left(\left(1+\tau_{R,t}\right) R^k_{t+s}\right)^{\alpha} \left(\left(1+\tau_{Pay,t}\right) W_{t+s}\right)^{1-a}}{A_{t+s}^{1-\alpha}}$$

where $\widehat{\alpha} = (1-\alpha)^{\alpha-1} \alpha^{-\alpha}$. The capital labor ratio is:

$$\frac{v_t K_{t-1}(i)}{N_t(i)} = \frac{\alpha}{1-\alpha} \frac{\left(1+\tau_{Pay,t}\right) W_t}{\left(1+\tau_{R,t}\right) R_{K,t}}$$

Calvo pricing of intermediates that also index to inflation if cannot reoptimize

$$P_{H,t+1|t}(i) = P^*_{H,t} \mathcal{I}^p_{t+1|t}$$

and if they can reoptimize

$$E_t \sum_{s=0}^{\infty} (\beta\theta_p)^s \frac{\Lambda_{t+s}}{\Lambda_t} \left[P^*_{H,t} \mathcal{I}^p_{t+s|t} Y_{t+s|t} - MC_{t+s|t} Y_{t+s|t}\right]$$

subject to product demand

$$Y_{t+s|t}(i) = \left(\frac{P^*_{H,t} \mathcal{I}^p_{t+s|t}}{P_{H,t+s}}\right)^{-\epsilon_p} Y_{t+s}$$

The otpimal price is

$$\frac{P^*_{H,t}}{P_{H,t}} = \frac{\epsilon_p}{\epsilon_p - 1} \frac{\mathcal{F^P}_t}{\mathcal{G^P}_t}$$

where $\mathcal{F^P}_t = \Lambda_t Y_t \frac{MC_{t|t}}{P_{C,t}} \frac{P_{C,t}}{P_{H,t}} + \beta\theta_p E_t \left(\frac{P_{H,t} \mathcal{I}^p_{t+1|t}}{P_{H,t+1}}\right)^{-\epsilon_p} \mathcal{F^P}_{t+1}$ and $\mathcal{G^P}_t = \Lambda_t Y_t + \beta\theta_p E_t \left(\frac{\mathcal{I}^P_{t+1|t} P_{H,t}}{P_{H,t+1}}\right)^{1-\epsilon_p} \mathcal{G^P}_{t+1}$.

Notice if price are flexibles we get the usual

$$\left(\frac{P^*_{H,t}}{P_{H,t}}\right)^{flex} = \frac{\epsilon_p}{\epsilon_p - 1} \frac{MC_t}{P_{H,t}}$$

The aggregate output price

$$P_{H,t} = \left[\int_0^1 P_t(i)^{1-\epsilon_p} di\right]^{\frac{1}{1-\epsilon_p}}.$$

Final Goods Firms

Final goods firms combine domestic and imported goods to produce two final non-tradable goods

Consumption goods

$$Y_{C,t} = \left[(\varphi_C)^{\frac{1}{\epsilon_C}} \left(C_{H,t}\right)^{\frac{\epsilon_C - 1}{\epsilon_C}} + (1-\varphi_C)^{\frac{1}{\epsilon_C}} \left(C_{F,t}\right)^{\frac{\epsilon_C - 1}{\epsilon_C}} \right]^{\frac{\epsilon_C}{\epsilon_C - 1}}$$

where $C_{H,t} = \left(\int_0^1 C_{H,t}(i)^{1-\frac{1}{\epsilon_p}} di \right)^{\frac{\epsilon_p}{\epsilon_p - 1}}$ and $C_{F,t} = \left(\int_0^1 C_{F,t}(i)^{1-\frac{1}{\epsilon_p}} di \right)^{\frac{\epsilon_p}{\epsilon_p - 1}}$

The optimal allocation of consumptionin domestic goods

$$C_{H,t} = \varphi_C \left(\frac{P_{H,t}}{P_{C,t}} \right)^{-\epsilon_C} Y_{C,t}$$

and across goods

$$C_{H,t}(i) = \left(\frac{P_{H,t}(i)}{P_{H,t}} \right)^{-\epsilon_p} C_{H,t}$$

The optimal allocation of consumption in foreign goods

$$C_{F,t}(i) = \left(\frac{P_{F,t}(i)}{P_{F,t}} \right)^{-\epsilon_p} C_{F,t}$$

$$C_{F,t} = (1-\varphi_C) \left(\frac{P_{F,t}}{P_{C,t}} \right)^{-\epsilon_C} Y_{C,t}$$

The CPI is

$$P_{C,t} \equiv \left(\varphi_C \left(P_{H,t}\right)^{1-\epsilon_C} + (1-\varphi_C) \left(P_{F,t}\right)^{1-\epsilon_C} \right)^{\frac{1}{1-\epsilon_C}}$$

Investment goods

$$Y_{I,t} = \left[(\varphi_I)^{\frac{1}{\epsilon_I}} \left(I_{H,t}\right)^{\frac{\epsilon_I - 1}{\epsilon_I}} + (1-\varphi_I)^{\frac{1}{\epsilon_I}} \left(I_{F,t}\right)^{\frac{\epsilon_I - 1}{\epsilon_I}} \right]^{\frac{\epsilon_I}{\epsilon_I - 1}}$$

where $I_{H,t} = \left(\int_0^1 I_{H,t}(i)^{1-\frac{1}{\epsilon_p}} di \right)^{\frac{\epsilon_p}{\epsilon_p - 1}}$ and $I_{F,t} = \left(\int_0^1 I_{F,t}(i)^{1-\frac{1}{\epsilon_p}} di \right)^{\frac{\epsilon_p}{\epsilon_p - 1}}$.

The optimal allocation of investment in domestic goods

$$I_{H,t} = \varphi_I \left(\frac{P_{H,t}}{P_{I,t}} \right)^{-\epsilon_I} Y_{I,t}$$

and across goods

$$I_{H,t}(i) = \left(\frac{P_{H,t}(i)}{P_{H,t}} \right)^{-\epsilon_p} I_{H,t}$$

The optimal allocation of investment in foreign goods

$$I_{F,t} = (1-\varphi_I) \left(\frac{P_{F,t}}{P_{I,t}} \right)^{-\epsilon_I} Y_{I,t}$$

and across goods

$$I_{F,t}(i) = \left(\frac{P_{H,t}(i)}{P_{H,t}} \right)^{-\epsilon_p} I_{F,t}$$

The Investment deflator

$$P_{I,t} \equiv \left(\varphi_I \left(P_{H,t}\right)^{1-\epsilon_I} + (1-\varphi_I) \left(P_{F,t}\right)^{1-\epsilon_I} \right)^{\frac{1}{1-\epsilon_I}}$$

Government spending

Public consumption is only made in domestic goods.

$$G_{H,t}(i) = \left(\frac{P_{H,t}(i)}{P_{H,t}} \right)^{-\epsilon_p} G_{H,t}$$

International Prices

The terms of trade are

$$\mathcal{S}_t \equiv \frac{P_{F,t}}{P_{H,t}}$$

the real eschange rate is

$$\mathcal{Q}_t \equiv \frac{P_t^*}{P_t}$$

International demand

The foreign countries only buy consumption goods from home

$$C_{H,t}^*(i) = \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\epsilon_p} C_{H,t}^*$$

Government

Goverment budget constraint

$$\Psi_{G,t} B_{G,t}^s + (\tau_{K,t} + \tau_{R,t}) R_{K,t} K_{t-1} + (\tau_{N,t} + \tau_{P,t}) W_t N_t + \tau_{C,t} P_{C,t} C_t = B_{G,t-1}^s + P_{H,t} G_t + T_t^R + T_t^{NR} + \tau_t^U W_t U_t$$

Portfolio decisions

Solving for international portfolio choice is beyond the scope of this model. (e.g., see discussion in Devereux and Sutherland, 2008). We simply assume that domestic private and government are perfect substitutes (no domestic financial frictions) and that households hold and exogenous fraction ω_t of their portfolio in foreign assets which are also perfect substitutes. In practice the distinction between domestic and foreign assets is irrelant in our model and was introduce to favour future extensions.

Common monetary policy and External premium

The model assume a single currency area where the foreign short term interest rate i_F is exogenous and determines the riskless foreign asset price $\Psi_F = \frac{1}{1+i_F}$. The domestic interest rate is equal to the exogenous polict rare plus a premium that depends on the external investment position $i_H = i_F + \rho\,(b_H - b_G + b_F - b^*)$. $\Psi_H = \frac{1}{1+i_F+\rho(b_H-b_G+b_F-b^*)}$

$$\Psi_H = \Psi_F$$

Equilibrium

The domestic goods market equilibrium is obtained bu aggregating

$$Y_{H,t}(i) = C_{H,t}(i) + I_{H,t}(i) + G_{H,t}(i) + C_{H,t}^*(i)$$

which gives

$$\left(\int_0^1 Y_{H,t}(i)^{1-\frac{1}{\epsilon_p}} di\right)^{\frac{\epsilon_p}{\epsilon_p-1}} = \left\{\int_0^1 \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\epsilon_p} \left(\varphi_C \left(\frac{P_{H,t}}{P_{C,t}}\right)^{-\epsilon_C} Y_{C,t} + \varphi_I \left(\frac{P_{H,t}}{P_{I,t}}\right)^{-\epsilon_I} Y_{I,t} + G_{H,t} + C_{H,t}^*\right)^{1-\frac{1}{\epsilon_p}} di\right\}^{\frac{\epsilon_p}{\epsilon_p-1}}$$

or

$$Y_{H,t} = C_{H,t} + I_{H,t} + G_{H,t} + C_{H,t}^*$$

Aggregate supply

$$\left[\int_0^1 \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\epsilon_p} di\right] Y_{H,t} = (v_t K_{t-1})^{\alpha} (A_t N_t)^{1-\alpha}$$

$$\Delta_{p,t} Y_{H,t} = (v_t K_{t-1})^{\alpha} (A_t N_t)^{1-\alpha}$$

where price dispersion evolves according to

$$\begin{aligned}\Delta_{p,t} &= \int_0^1 \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\epsilon_p} di \\ &= \theta_p \int_0^1 \left(\frac{P_{H,t-1}(i)\mathcal{I}^p_{t|t-1}}{P_{H,t}}\frac{P_{H,t-1}}{P_{H,t-1}}\right)^{-\epsilon_p} di + (1-\theta_p)\left(\frac{P^*_{H,t}}{P_{H,t}}\right)^{-\epsilon_p} \\ &= \theta_p \Delta_{p,t-1}\left(\frac{P_{H,t-1}\mathcal{I}^p_{t|t-1}}{P_{H,t}}\right)^{-\epsilon_p} + (1-\theta_p)\left(\frac{\epsilon_p}{\epsilon_p - 1}\frac{\mathcal{F}^{\mathcal{P}}{}_t}{\mathcal{G}^{\mathcal{P}}{}_t}\right)^{-\epsilon_p}\end{aligned}$$

We see that all firms have the same K/N, aggregating

$$\upsilon_t K_{t-1} = N_t \frac{\alpha}{1-\alpha}\frac{(1-\tau_{Pay,t})\, W_t}{(1-\tau_{R,t})\, R_{K,t}}.$$

Labor supply (for welfare)

$$\int_0^1 N_t(j)dj = \left[\int_0^1 \left(\frac{W_t(j)}{W_t}\right)^{-\epsilon_w} dj\right] N_t$$

$$\int_0^1 N_t(j)dj = \Delta_{w,t} N_t$$

Labor foce participation

$$\frac{W^*_t(j)}{P_{C,t}} = \frac{\chi_t L(j)^{\phi} Z_t}{\Lambda_t\left(1-\tau^N_t\right)}$$

so that

$$L_t = \left(\frac{\Lambda_t\left(1-\tau^N_t\right)}{\chi_t Z_t}\frac{W_t}{P_{C,t}}\right)^{\frac{1}{\phi}} \int_0^1 \left(\frac{W^*_t(j)}{W_t}\right)^{\frac{1}{\phi}} dj$$

and wage dispersion is

$$\begin{aligned}\Delta_{w,t} &= \int_0^1 \left(\frac{W^*_t(j)}{W_t}\right)^{\frac{1}{\phi}} dj \\ &= \theta_w \int_0^1 \left(\frac{W_{t-1}(j)\mathcal{I}^W_{t|t-1}}{W_{t-1}}\frac{W_{t-1}}{W_t}\right)^{\frac{1}{\phi}} dj + (1-\theta_w)\left(\frac{W_t}{P_{C,t}}\right)^{-\frac{1}{\phi}}\left(\frac{W^*_t}{P_{C,t}}\right)^{\frac{1}{\phi}} \\ &= \theta_w \Delta_{w,t-1}\left(\frac{\mathcal{I}^W_{t|t-1}}{\Pi_{w,t-1}}\right)^{\frac{1}{\phi}} + (1-\theta_w)\left(\frac{W_t}{P_{C,t}}\right)^{-\frac{1}{\phi}}\left(\frac{W^*_t}{P_{C,t}}\right)^{\frac{1}{\phi}}\end{aligned}$$

The equilibrium in the final consumption sector

$$\mu C^R_t + (1-\mu)\, C^{NR}_t + C^U_t + \mathcal{C}\left(\upsilon_t\right) K_{t-1} = Y_{C,t}$$

and in the investment sector:

$$I_t = Y_{I,t}$$

Aggregating Households, Firrms and Government budget constraint we obrain the country budget constraint:

$$\frac{\Psi_{G,t}}{P_{C,t}}\left(B_{G,t} - B^s_{G,t}\right) + \sum_{k\in J\backslash G}\frac{\Psi_{k,t}}{P_{C,t}}B_{k,t} = \sum_{k\in J\backslash G}\frac{P_{C,t-1}}{P_{C,t}}\frac{B_{k,t-1}}{P_{C,t-1}} + \frac{P_{C,t-1}}{P_{C,t}}\frac{\left(B_{G,t-1} - B^s_{G,t-1}\right)}{P_{C,t-1}} + \frac{P_{H,t}}{P_{C,t}}\left[C_{H^*,t} - \mathcal{S}_t\left(C_{F,t} + I_{F,t}\right)\right]$$

Fiscal Rules

Various types of common rules such as

- the nominal balance fiscal rule

$$\frac{Def_t}{Y_t} = d^* - \lambda\left(\frac{B_{H,t-1}}{Y_{t-1}} - b^*_H\right)$$

- the structural balance fiscal rule

$$\frac{Def_t}{Y_t} = d^* - \alpha\left(\frac{Y_t - Y_t^n}{Y_t^n}\right) - \lambda\left(\frac{B_{H,t-1}}{Y_{t-1}} - b_H^*\right)$$

- Expenditure growth rule

$$\frac{G_{H,t}}{G_{H,t-1}} - 1 = \alpha\left(\frac{Y_t^n}{Y_{t-1}^n} - 1\right) - \lambda\left(\frac{B_{H,t-1}}{Y_{t-1}} - b_H^*\right)$$

- Modified Expenditure growth rule $ME_t = P_{H,t}G_{H,t} - i_{H,t}B_{H,t} - UB_t$

$$\frac{ME_{,t}}{ME_{t-1}} - 1 = \alpha\left(\frac{Y_t^n}{Y_{t-1}^n} - 1 + \Pi^*\right) - \hat{\lambda}\left(\frac{B_{H,t-1}}{Y_{t-1}} - b_H^*\right)$$

Steady State

S is exogenous and pins down the relative prices:

$$\mathcal{P}_{CH} \equiv \frac{P_C}{P_H} = \left(\varphi_C + (1-\varphi_C)\,\mathcal{S}^{1-\epsilon_C}\right)^{\frac{1}{1-\epsilon_C}}$$

$$\mathcal{P}_{IH} \equiv \frac{P_I}{P_H} = \left(\varphi_I + (1-\varphi_I)\,\mathcal{S}^{1-\epsilon_I}\right)^{\frac{1}{1-\epsilon_I}}$$

$$\mathcal{P}_{IC} \equiv \frac{P_I}{P_C} = \frac{\left(\varphi_I + (1-\varphi_I)\,\mathcal{S}^{1-\epsilon_I}\right)^{\frac{1}{1-\epsilon_I}}}{\left(\varphi_C + (1-\varphi_C)\,\mathcal{S}^{1-\epsilon_C}\right)^{\frac{1}{1-\epsilon_C}}}$$

this pins down the investment decision

$$Q = \frac{\mathcal{P}_{IC}}{\mathcal{D}}$$

and the return on capital

$$r_K = Q\frac{(1-\beta(1-\delta))}{\beta(1-\tau_K)}$$

from the optimal price setting we have the labor demand

$$w = \left(\frac{\epsilon_p}{\epsilon_p - 1}\widehat{\alpha}\frac{r_K^\alpha}{A}\mathcal{P}_{CH}\right)^{\frac{1}{\alpha-1}}$$

having the input prices gives the capital labor ratio

$$\frac{K}{N} = \frac{\alpha}{1-\alpha}\frac{(1+\tau_{Pay})\,w}{(1+\tau_R)\,r_K}$$

No income effect on the labor supply (simpler)

If labor supply does not depend on income say $\chi = \bar{\chi}\Lambda$ then from

$$w = \frac{\epsilon_w}{\epsilon_w - 1}\frac{\bar{\chi}\mathcal{N}^\phi Z}{(1-\tau^N)}$$

we get N. Then we have K and Y_H. The from and dometic goods demand

$$I = \frac{\delta}{\mathcal{D}}K$$

$$Y_H = \left(\varphi_C\mathcal{P}_{C\mathcal{H}}{}^{\epsilon_C}C + \varphi_I\mathcal{P}_{IH}{}^{\epsilon_I}I + G_H + C_H^*\right)$$

we get C.

Fiscal instrument: G_H

If G_H is exogenous we have the equilibrium.

$$C = \frac{Y_H - \varphi_I\mathcal{P}_{IH}{}^{\epsilon_I}I - G_H - C_H^*}{\varphi_C\mathcal{P}_{C\mathcal{H}}{}^{\epsilon_C}}$$

Fiscal instrument: T

If G_H is endogenous and $\frac{Def}{Y_H}$ is exogenous (together with T^R and T^{NR}) you also need the government deficit definition:

$$\tau_C \frac{C}{Y_H} - \frac{G_H + T^R + T^{NR}}{Y_H} + \frac{Def}{Y_H} = \tau_u \frac{U}{Y_H} w - (\tau_K + \tau_R) r_K \frac{K}{Y_H} - (\tau_N + \tau_{Pay}) w \frac{N}{Y_H}$$

where $\frac{U}{Y_H} = \left(\left(\frac{\epsilon_w}{\epsilon_w - 1} \right)^{\frac{1}{\phi}} - 1 \right) \frac{N}{Y_H}$.

Fiscal instrument: $\frac{b_G}{Y_H}$

If $\frac{b_G}{Y_H}$ is exogenous

$$\left(\Psi_H - \frac{1}{\Pi_c} \right) \frac{b_G}{Y_H} = \frac{Def}{Y_H}$$

$\frac{Def}{Y_H}$ is pinned down because Ψ_H is given by

$$\Psi_H + \rho (b_H - b_G + b_F - b^*) = \Psi_F$$

external balance

$$C_{H^*} - \mathcal{S} (C_F + I_F) = \mathcal{P}_{CH} \left[\left(\Psi_H - \frac{1}{\Pi_c} \right) (b_H - b_G) + \left(\Psi_F - \frac{1}{\Pi_c} \right) b_F \right]$$

and portfolio decisions

$$b_F = \omega b_H$$

Considering that $C_F = (1 - \varphi_C) \left(\frac{\mathcal{P}_{CH}}{\mathcal{S}} \right)^{\epsilon_C} C$ and $I_F = (1 - \varphi_I) \left(\frac{\mathcal{P}_{CH}}{\mathcal{S}} \right)^{\epsilon_I} I$.

Think about the Maastricht Rule that makes both $\frac{b_G}{Y_H}$ and $\frac{Def}{Y_H}$ exogenous then the debt accumulation equation determines

$$\left(\Psi_H - \frac{1}{\Pi_c} \right) \frac{b_G}{Y_H} = \frac{Def}{Y_H}$$

they might be incosistent with

$$\Psi_H + \rho (b_H - b_G + b_F - b^*) = \Psi_F$$

unless b^* becomes endogenous.

If labor supply depends on income effect N, K, Y_H are determined simultaneously to C and using

$$w = \frac{\epsilon_w}{\epsilon_w - 1} \frac{\chi \mathcal{N}^{\phi} Z}{\Lambda (1 - \tau^N)}$$

$$\Lambda = \left(1 + \tau^C\right)^{-1} (C + \alpha_G G_H)^{-\sigma} (1 - \beta \mathfrak{h})$$

Log-linear model

$$\lambda_t + \frac{\tau_C}{1 + \tau_C} \hat{\tau}_{C,t} = \frac{(z_t - \sigma \tilde{c}_t) - \beta \mathfrak{h} E_t (z_{t+1} - \sigma \tilde{c}_{t+1})}{1 - \beta \mathfrak{h}} \tag{1}$$

$$\psi_t = E_t [\lambda_{t+1} - \lambda_t - \pi_{C,t+1}] \tag{2}$$

$$i_{H,t} = -\frac{\psi_{H,t}}{\Psi_H} \tag{3}$$

$$q_t = E_t \lambda_{t+1} - \lambda_t + \beta \frac{(1 - \tau_K) r_K}{Q} E_t r_{K,t+1} - \beta \frac{\tau_K r_K}{Q} E_t \hat{\tau}_{K,t+1} + \beta \frac{(1 - \tau_K) r_K}{Q} E_t \upsilon_{t+1} - \beta \frac{\mathcal{C}'(\upsilon)}{Q} E_t \upsilon_{t+1} + \beta (1 - \delta) E_t q_{t+1}$$

using steady state values

$$q_t = E_t \lambda_{t+1} - \lambda_t + (1 - \beta (1 - \delta)) E_t r_{K,t+1} - \tau_K \frac{(1 - \beta (1 - \delta))}{(1 - \tau_K)} E_t \hat{\tau}_{K,t+1} + (1 - \beta (1 - \delta)) E_t \upsilon_{t+1} + \beta (1 - \delta) E_t q_{t+1} \tag{4}$$

$$p_{IC,t} = \frac{Q\mathcal{D}}{P_{IC}} (q_t + d_t) - \frac{Q\mathcal{D}}{P_{IC}} \phi_I (i_t - i_{t-1}) + \beta \frac{Q\mathcal{D}}{P_{IC}} \phi_I E_t [(i_{t+1} - i_t)]$$

$\frac{Q\mathcal{D}}{P_{IC}} = 1$ in steady state

$$p_{IC,t} = q_t + d_t - \xi_I (i_t - i_{t-1}) + \beta \xi_I E_t [(i_{t+1} - i_t)] \tag{5}$$

$$r_{K,t} - \frac{\tau_K}{1+\tau_K}\hat{\tau}_{K,t} = \frac{\mathcal{C}''(\upsilon)\,\upsilon}{\mathcal{C}'(\upsilon)}\hat{\upsilon}_t$$

$$\mathcal{C}'(\upsilon_t) = (1-\tau_K)\frac{R_K}{P_C}\left(e^{(\phi_v(\nu-1))}\right)$$

$\mathcal{C}''(\upsilon_t) = \phi_v(1-\tau_K)\frac{R_K}{P_C}\left(e^{(\phi_v(\nu-1))}\right)$

$$r_{K,t} - \frac{\tau_K}{1+\tau_K}\hat{\tau}_{K,t} = \phi_v\hat{\upsilon}_t \tag{6}$$

combining 7,8,9,10

$$\pi^w_t - \iota^w_{t|t-1} = \beta E_t\left(\pi^w_{t+1} - \iota^w_{t+1|t}\right) - \frac{(1-\theta_w)(1-\beta\theta_w)}{(1+\phi\epsilon_w)\,\theta_w}\left(w_t - mrs_t - \mu^n_{w,t}\right) \tag{7}$$

$$\mu_{w,t} = w_t - mrs_t \tag{8}$$

$$mrs_t = \phi n_t + z_t + \hat{\chi}_t - \lambda_t - \frac{\tau_N}{(1-\tau_N)}\hat{\tau}_{N,t} \tag{9}$$

$$w_t = \phi l_t + z_t + \hat{\chi}_t - \lambda_t - \frac{\tau_N}{(1-\tau_N)}\hat{\tau}_{N,t} \tag{10}$$

$$u_t = l_t - n_t \tag{11}$$

$$mc_t = \alpha\left(r_{K,t} + \frac{\tau_R}{(1+\tau_R)}\hat{\tau}_{R,t}\right) + (1-\alpha)\left(w_t + \frac{\tau_{Pay}}{(1+\tau_{Pay})}\hat{\tau}_{Pay,t}\right) - (1-\alpha)\,a_t \tag{12}$$

coombining 12,13,14,15

$$\pi_{H,t} - \iota^p_{t|t-1} = \frac{(1-\theta_p)(1-\beta\theta_p)}{\theta_p}\left(mc_t + p_{CH,t} + \mu^n_{p,t}\right) + \beta E_t\left(\pi_{H,t+1} - \iota^p_{t+1|t}\right) \tag{13}$$

$$\mu_{p,t} = -mc_t - p_{CH,t} \tag{14}$$

$$p_{CH,t} = (1-\varphi_C)\,s_t \tag{15}$$

$$c_{H,t} = \epsilon_C p_{CH,t} + c_t \tag{16}$$

$$c_{F,t} = \epsilon_C\left(p_{CH,t} - s_t\right) + c_t \tag{17}$$

$$p_{IH,t} = (1-\varphi_I)\,s_t \tag{18}$$

$$i_{H,t} = \epsilon_I p_{IH,t} + i_t \tag{19}$$

$$i_{F,t} = \epsilon_I\left(p_{IH,t} - s_t\right) + i_t \tag{20}$$

$$y_{H,t} = \frac{C_H}{Y_H}c_{H,t} + \frac{I_H}{Y_H}i_{H,t} + \frac{G_H}{Y_H}g_{H,t} + \frac{C^*_H}{Y_H}c^*_{H,t} \tag{21}$$

$$y_{H,t} = \alpha\left(\hat{\nu}_t + k_{t-1}\right) + (1-\alpha)\left(a_t + n_t\right) \tag{22}$$

$$c_t = \frac{C^U}{C}c^U_t + \frac{C^R}{C}\mu c^R_t + \frac{C^{NR}}{C}(1-\mu)\,c^{NR}_t + \frac{\mathcal{C}'(\upsilon)\,K}{C}\hat{\nu}_t \tag{23}$$

$$(1+\tau_C)\,C^U c^U_t + \tau_C C^U\hat{\tau}_{C,t} = \tau_u\frac{W}{P_C}U\left(\tau_{u,t} + w_t + u_t\right)$$

$$(1+\tau_C)\,C^{NR}c^{NR}_t + \tau_C C^{NR}\hat{\tau}_{C,t} = (1-\tau_N)\frac{W}{P_C}N\left(w_t + n_t\right) - wN\frac{\tau_N}{(1-\tau_N)}\hat{\tau}_{N,t} + t^{TR}_t \tag{24}$$

$$\tilde{c}_t = \frac{C}{\tilde{C}}c_t + \alpha_G\frac{G_H}{\tilde{C}}g_{h,t} \tag{25}$$

$$k_t = (1-\delta)\,k_{t-1} + \delta\left(d_t + i_t\right) \tag{26}$$

$$\hat{\nu}_t + k_{t-1} - n_t = \left(w_t + \frac{\tau_{Pay}}{(1+\tau_{Pay})}\hat{\tau}_{Pay,t}\right) - \left(r_{K,t} + \frac{\tau_R}{(1+\tau_R)}\hat{\tau}_{R,t}\right) \tag{27}$$

$$\psi_{H,t} + b_{G,t} = \frac{\Psi_H^{-1}}{\Pi_C}b_{G,t-1} + \frac{Def}{\Psi_H B_G}def_t$$

$$Defdef_t = G_H g_{H,t} + T^R t_t^R + T^{NR} t_t^{NR} + U \frac{W}{P_C} \tau_u \left(\hat{\tau}_{u,t} + w_t + u_t\right) - \left(\tau_K + \tau_R\right) \frac{R_K}{P_C} K \left(r_{K,t} + k_t\right) - \frac{R_K}{P_C} K \left(\tau_K \hat{\tau}_{K,t} + \tau_R \hat{\tau}_{R,t}\right) \tag{28}$$

$$- \left(\tau_N + \tau_P\right) \frac{W}{P_C} N \left(w_t + n_t\right) - \frac{W}{P_C} N \left(\tau_N \hat{\tau}_{N,t} + \tau_{Pay} \hat{\tau}_{Pay,t}\right) - \tau_C C \left(\hat{\tau}_{C,t} + c_t\right)$$

$CA_{SS} = \frac{\Psi_H}{P_C} \left(B_H - B_G\right) + \frac{\Psi_F}{P_C} B_F$

$$CA_{SS} \left(\Psi_H \left(B_H - B_G\right) \psi_{H,t} + \Psi_H \left(B_H b_{H,t} - B_G b_{G,t}\right) + \Psi_F B_F \left(\psi_{F,t} + b_{F,t}\right)\right) = \frac{B_H}{\Pi_C} b_{H,t-1} - \frac{B_G}{\Pi_C} b_{G,t-1} + \frac{B_F}{\Pi_C} b_{F,t-1} - \frac{\left(B_H - B_G + B_F\right)}{\Pi_C} \pi_{C,t} + \left[C_{H^*} - \mathcal{S}\left(C_F + I_F\right)\right] p_{CH,t} + C_{H^*} c^*_{H,t} - \left(C_F + I_F\right) s_t - \left(C_F c_{F,t} + I_F i_{F,t}\right) \tag{29}$$

$$b_{F,t} = \omega_t + b_{H,t} \tag{30}$$

$$i_{H,t} = i_{F,t} + \xi_t \left(B_H b_{H,t} - B_G b_{G,t} + B_F b_{F,t}\right) + \xi_t \tag{31}$$

$$\Delta s_t = \pi_{F,t} - \pi_{H,t} \tag{32}$$

$$c^*_{H,t} = \epsilon_c \left(p_{CH,t} + reer_t\right) + c^*_t \tag{33}$$

$$def_t - y_{h,t} =$$

exogenous

$$z_t = \rho_z z_{t-1} + \eta_{z,t}$$

$$a_t = \rho_z a_{t-1} + \eta_{a,t}$$

$$\mu^n_{p,t} = \rho_p \mu^n_{p,t-1} + \eta_{p,t}$$

$$\mu^n_{w,t} = \rho_w \mu^n_{w,t-1} + \eta_{w,t}$$

$$d_t = \rho_{zd} d_{t-1} + \eta_{d,t}$$

$$c^*_t = \rho_{c^*} c^*_{t-1} + \eta_{c^*,t}$$

$$\chi_t = \rho_\chi \chi_{t-1} + \eta_{\chi,t}$$

$$\xi_t = \rho_\xi \xi_{t-1} + \eta_{\xi,t}$$

natural levels WITH FLEX PRICES

$$mc_t + p_{CH,t} = -\mu^n_{p,t}$$

$$w_t - mrs_t = \mu^n_{w,t}$$

$$mc_t = \alpha \left(r_{K,t} + \frac{\tau_R}{\left(1 + \tau_R\right)} \hat{\tau}_{R,t}\right) + \left(1 - \alpha\right) \left(w_t + \frac{\tau_{Pay}}{\left(1 + \tau_{Pay}\right)} \hat{\tau}_{Pay,t}\right) - \left(1 - \alpha\right) a_t$$

$$\left(r_{K,t} + \frac{\tau_R}{\left(1 + \tau_R\right)} \hat{\tau}_{R,t}\right) = \frac{mc_t}{\alpha} - \frac{1 - \alpha}{\alpha} \left(w_t + \frac{\tau_{Pay}}{\left(1 + \tau_{Pay}\right)} \hat{\tau}_{Pay,t}\right) + \frac{1 - \alpha}{\alpha} a_t$$

$$\hat{\nu}_t + k_{t-1} - n_t = \left(w_t + \frac{\tau_{Pay}}{\left(1 + \tau_{Pay}\right)} \hat{\tau}_{Pay,t}\right) - \left(r_{K,t} + \frac{\tau_R}{\left(1 + \tau_R\right)} \hat{\tau}_{R,t}\right)$$

$$y_{H,t} = n_t + \alpha \left(\hat{\nu}_t + k_{t-1} - n_t\right) + \left(1 - \alpha\right) a_t$$

$$y_{H,t} = n_t + \alpha \left(\left(w_t + \frac{\tau_{Pay}}{\left(1 + \tau_{Pay}\right)} \hat{\tau}_{Pay,t}\right) - \left(r_{K,t} + \frac{\tau_R}{\left(1 + \tau_R\right)} \hat{\tau}_{R,t}\right)\right) + \left(1 - \alpha\right) a_t$$

$$y_{H,t} = n_t + w_t + \frac{\tau_{Pay}}{\left(1 + \tau_{Pay}\right)} \hat{\tau}_{Pay,t} - mc_t$$

$$y_{H,t} = n_t + \mu^n_{w,t} + mrs_t + \frac{\tau_{Pay}}{\left(1 + \tau_{Pay}\right)} \hat{\tau}_{Pay,t} + \mu^n_{p,t} + p_{CH,t}$$

$$mrs_t = \phi n_t + z_t + \hat{\chi}_t - \lambda_t - \frac{\tau_N}{\left(1 - \tau_N\right)} \hat{\tau}_{N,t}$$